Adaptive Parzen Windowing on Mutual information for Intermodal Non-rigid Image Registration

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***Abstract*—Mutual information is widely used as a similarity metric for inter-modality image registration because of its accuracy compared to other conventional metrics. However, mutual information may lead to false registration result when signal to noise ratios of given images are small or when local statistics differ from global statistics. Therefore, we suggest a novel adaptive Parzen windowing method to increase the accuracy of computing mutual information. The proposed method adapts the window size to more accurately estimate the joint density histogram by ensuring at least a minimum number of local counts. Simulation results show that our adaptive Parzen windowing method outperforms not only conventional mutual information as computed via standard Parzen windowing but also other similarity metrics designed to improve accuracy of mutual information.**

I. Introduction

Mutual information is a well-known similarity measure for registration of medical images. It is widely used especially for inter-modality image registration because of its robustness compared to other conventional metrics such as cross-correlation and sum of squared intensity differences. However, several authors [1]–[3] have reported that conventional mutual information may lead to false registration depending on overlap size. After Studeholme [1] suggested normalized mutual information as an overlap invariant generalization of mutual information, August [2] and Cahill [3] revisited the problem in rigid registration and suggested using background or non-overlap voxels statistics to compensate for the inaccuracy of mutual information due to changing overlap size.

High degree of freedom (DOF) non-rigid image registration is a challenging task. Applying mutual information to non-rigid image registration is a challenge because local intensity changes caused by imaging distortion may be poorly reflected in global statistics. Others [4]–[6] have approached this problem by introducing spatial information as another channel of information, but the complexity of such algorithms increase their computational time.

In this paper, we describe both of these issues (overlap size and local/global statistics mismatch) as a problem of signal to noise ratio (SNR) of overlapping regions that can be solved by efficiently filtering the estimate of the joint density histogram (JDH). Then we propose a novel adaptive Parzen windowing of JDH to improve the accuracy of mutual information estimation as a similarity metric. The proposed method adaptively chooses local window sizes for estimating the JDH based on the local number of samples.

We use a single 2D chest CT scan to synthesize multimodal images and validate our method by a simple one DOF rigid registration. Next, we randomly warp 2D chest CT scans using thin-plate spline to synthesize an image with known ground truth and validate our method using 32 DOF non-rigid registration. Simulation results show that our proposed method outperforms other similarity metrics by improving registration accuracy with short computational time.

Section 2 interprets the problems using mutual information as a similarity metric. Section 3 gives a detailed description of our method. Section 4 presents simulation results and discussion. Section 5 gives conclusions and future works.

II. Interpretation of the Problems

The nature of mutual information can be interpreted based on dispersion of JDH [7]. The more clustered JDH is, the higher mutual information is, and the better the two images are regarded to be registered. Typically JDH is estimated by conventional Parzen windowing even though the SNRs of the inputs may vary spatially. An extreme example of such variation occurs in partially overlapping datasets where noisy non-overlapping regions of poor signal strength yield sparsely distributed Kronecker delta functions in the input's unfiltered JDH which are interpreted as regions of high mutual information. In the optimization of such faulty mutual information estimates the datasets are typically pulled further apart. Picking Parzen window functions to be sufficiently wide to prevent such behavior limits the sensitivity of the registration only to large, global features, and thus negatively affects the algorithm's abilities to perform more local, higher DOF, nonlinear registrations. Therefore, we propose an adaptive Parzen windowing method to adapt the window size to local SNR.

III. Method

Let $I_R(x_R)$ and $I_F(x_F)$ be intensities of image R and F. $g(x_R;\mu)$ is the transformation field where μ represents transformation parameters. JDH without any windowing can be defined as

$$h_g(r,f) \triangleq \sum_{x_R \in D_R} \delta(I_R(x_R)-r)\delta(I_F(g(x_R;\mu))-f) \quad (1)$$

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where $I_R : D_R \to 0, 1, \ldots, 2^K$ and $I_F : D_F \to 0, 1, \ldots, 2^L$.

$\delta(\cdot)$ is the Kronecker delta:

$$\delta(x) = \begin{cases} 1, & \text{if } x = 0, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

We define $w_{r,f} \in 1, 3, 5, ..., M$ to be size of adaptive 2D window for each JDH bin where M is the maximum window width available. We also define a threshold value determined by image size as T.

For each (r, f)-th bin, the window size is defined as follows.

Let $w_{r,f} = M$.

$$h'_g(r, f; \mu) \triangleq \sum_{i=-(w_{r,f}-1)/2}^{(w_{r,f}-1)/2} \sum_{j=-(w_{r,f}-1)/2}^{(w_{r,f}-1)/2} h_g(r + i, f + j; \mu) \tag{3}$$

$$= \sum_{x_R \in D_R} W(I_R(x_R) - r; h_g) W(I_F(g(x_R; \mu)) - f; h_g). \tag{4}$$

If $h'(r, f; \mu) \geq T$ and $w_{r,f} > 1$, then decrease $w_{r,f} = w_{r,f} - 2$ and compute (2). Run this loop until $h'(r, f; \mu) < T$ or $w_{r,f} = 1$. Then the final update of JDH bin is normalized as $h'_g(r, f; /mu) = h'_g(r, f; \mu)/w^2_{r,f}$.

The new JDH can be normalized as probability density function as

$$p_g(r, f; \mu) = \frac{h'_g(r, f; \mu)}{\sum_{r,f} h'_g(r, f; \mu)}. \tag{5}$$

A. Mutual Information with Adaptive Parzen Windowing

The mutual information $S(\mu)$ of two images can be computed as

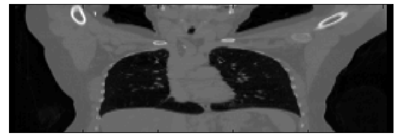
$$S(\mu) = \sum_{r,f} p_g(r, f; \mu) log \frac{p_g(r, f; \mu)}{p_g(r; \mu) p_g(f; \mu)}. \tag{6}$$

IV. Results and Discussion

To evaluate the proposed method we first demonstrate simple one degree rigid body transformation and compare with other similarity metrics [2], [3]. Next, we demonstrate high degree non-rigid registration and compare with other similarity metrics [2], [3], [5].

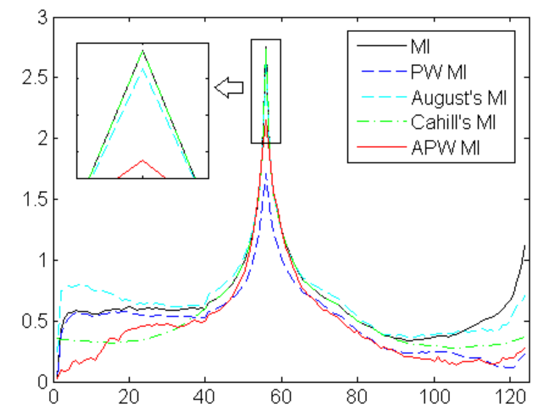
A. Experiment 1: Rigid Registration

This experiment used 2D chest CT image to visualize the proposed similarity metric (5) in simple one degree translation registration with known ground truth. Figure 1 (a) is the reference image with size 86×256 and (b) is the floating image with size 40×256. The floating image is a part of the reference image but having inverse intensity. The two images align exactly when the floating image translates 56 pixels upward in y-direction. Figure 1 (c) plots the similarity measure as a function of y-directional translation. The floating image translates from overlapping only with the bottom row of reference image to only with the top row of reference image. Five different similarity measures were compared. Conventional mutual information, conventional mutual information with Parzen window histogram estimation, August's [2] non-overlap involving mutual information, Cahill's [3] modified mutual information, and our proposed adaptive Parzen windowed mutual information. As shown in Figure1 (c), mutual information has limited capture range because of false local maximum when only one row from each image overlaps. Other four methods including our proposed method suppressed false local maximum at small overlap size trying to extend the capture range.



(a) reference image

(b) floating image



(c) similarity metrics changing according to 1D spatial translation.

Fig. 1. Synthesized example of inter-modality image registration based on rigid geometric transformation. (a) is the reference and (b) is the floating image. The floating image is a part of the reference image but having inverse intensity. The two images align exactly when the floating image translates 56 pixels in the y-direction. (c) shows five similarity metrics changing according to one dimensional translation (and the superimposed magnified peak), mutual information, mutual information with Parzen window histogram estimation, August's non-overlap involving mutual information, Cahill's modified mutual information, and our proposed adaptive Parzen windowed mutual information. A local maximum mutual information occurs where overlapping size of two image is minimum (only one row), when floating image translates 124 pixels in the y-direction. Other four methods including our proposed method suppressed false maximum at small overlap size. (d) shows the zoom

As shown in Table I the computational time is quite similar for this simple one dimensional translation example. Next section shows results from non-rigid registration.

TABLE I
Computational time (Matlab 2.93GHz CPU) for 1D translation

metric	time
MI w/o filtering	11.08 sec
PW MI	11.75 sec
August's MI	11.87 sec
Cahill's MI	11.14 sec
Proposed APW MI	12.03 sec

B. Experiment 2: Non-rigid Registration

This experiment used 2D chest CT image to validate performance of our proposed adaptive Parzen windowing method in high DOF non-rigid registration with known ground truth. 40 image pairs were first normalized in 256 gray level. Figure2 (a) is the reference image with size 256×256 and (b) is one of the 40 floating images with size 256×256. Floating images were random warps of the reference image using thin-plate spline transformation, so the ground truth transformation parameters were known. Sixteen gridded control points were used. The non-rigid registration process was conducted using thin-plate spline transformation. Figure2 (c) shows checkerboard fusion of registered images using proposed adaptive Parzen windowed mutual information.

We evaluated six different similarity metrics: Mutual information, mutual information with Parzen window histogram estimation, August's non-overlap involving mutual information [2], Cahill's modified mutual information [3], conditional mutual information [5], and our proposed adaptive Parzen windowed mutual information. For our proposed method, the maximum window width was chosen to be $W = 9$, and count threshold was chosen to be $T = 37$. Figure2 (d) and (e) show box plots of warping index (WI) and average intensity difference (AID) of registration results, respectively, representing registration error. Warping index is the difference between the calculated and true deformation field.

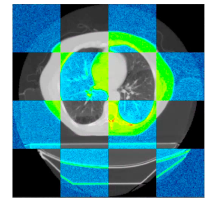
Figure2 (d) and (e) show that our proposed adaptive Parzen windowing has the best registration accuracy for this study. Note that, even for a perfect registration, the warping index might contain nonzero component within homogeneous regions. TableII shows the computational time of all six similarity metrics. Figure 2 and Table II, show that our proposed method offers high accuracy registration with reasonable computational time.

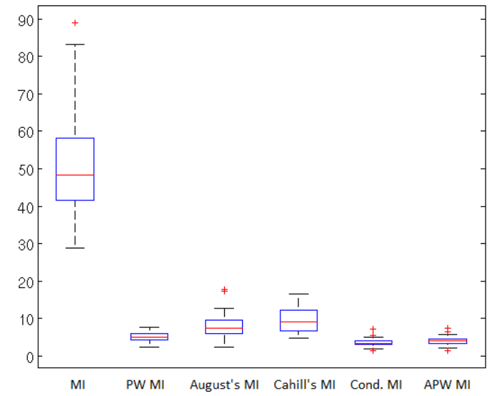
TABLE II
COMPUTATIONAL TIME (MATLAB 2.93GHZ CPU) FOR 16 DOF NON-RIGID REGISTRATION

metric	time
MI w/o filtering	110 sec
PW MI	700 sec
August's MI	4638 sec
Cahill's MI	656 sec
Conditional MI	3165 sec
Proposed APW MI	633 sec

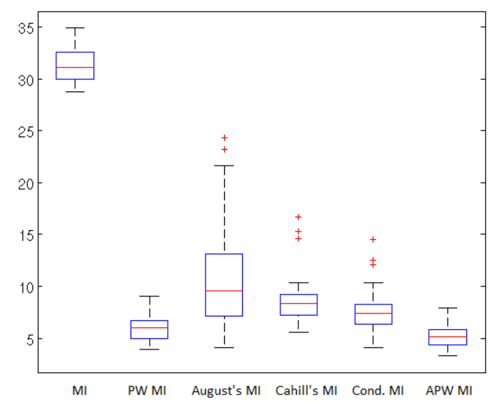

(a) reference image


(b) floating image


(c) checkerboard fusion



(d) warping index



(e) average intensity difference

Fig. 2. (a) is the reference image with size 256×256 and (b) is one of the 40 floating image with size 256×256. Floating images were random warp of the reference image using thin-plate spline transformation. Sixteen gridded control points were used. The non-rigid registration process was conducted using thin-plate spline transformation with 16 control points. (c) shows checkerboard fusion of registered images using proposed adaptive Parzen windowing on mutual information. (d) and (e) shows box plot of warping index(WI) and average intensity difference(AID) of registration results, respectively. Box plots shows that our proposed method performs highest registration accuracy.

C. Discussion

In Experiment 1, conventional mutual information showed false local maximum when overlap size of two images is the smallest. This is because when few number of samples exist, noise on either image can act like a cluster in JDH, ending up increasing mutual information. Our proposed adaptive Parzen window increases window size when there are few local counts so that noise is smoothed out, while turning off Parzen window when there are abundant local counts. This way, we could maintain sharp transition peak near actual alignment without over smoothing.

In Experiment 2, our adaptive filtering worked well in non-rigid situation because applying different filter width in different JDH pairs is interpreted as applying different filter in different spatial location. This way, local statistics can be reflected in calculating mutual information. Conditional mutual information [5] also does this by spatially encoding mutual information, but our proposed method requires less computational time.

V. Conclusions and Future Works

In this paper, we compared the limitation of mutual information similarity metrics based on dispersion of JDH and signal to noise ratio of two images. We showed with a simple one dimensional translation that our proposed adaptive Parzen windowing method prevents false local maximum of mutual information. We also showed that our proposed method improves registration accuracy for high degree non-rigid image registration. Compared to other state of the art metrics, our proposed method yields better registration accuracy in comparatively fast computational time. In our future work, we plan to investigate performance of similarity metrics in noise by measuring objective functions peak curvature, different maximum window size, and different count threshold parameter. We also plan to apply our proposed approach to 3D non-rigid image registration.

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